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Intensity based Model for CDS Spread with Time-Changed Levy Process

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Abstract

In credit default swap (CDS) pricing, it is important to find both flexible and tractable models. In this paper, by setting up a time-changed Levy process subordinated by Ornstein-Uhlenbeck we present an intensity based model for the CDS spread. We apply a stochastic volatility model driven by infinite activity Levy processes that is consistent with phenomenon observed in underlying asset. Some typical Levy process, aiming to capture the leptokurtic feature in asset returns and volatility clustering effect in returns variance are investigated. The asset value processes of these models are able to fit any valid CDS curve that would possibly be of both finite variations. Furthermore, we obtain a closed form formula for its survival function in terms of characteristic function of the time-changed Levy where the default is triggered by a predetermined barrier. This pricing formula is well calibrated on the CDS market by an intelligent global optimization search algorithm that improves the calibration quality of the fitted models.

Keywords: CDS spread, Time-changed Levy process, Differential Evolution algorithm
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1 Introduction

Credit default swaps (CDSs) have become the most popular credit derivatives in the past two decades. The CDS contract offers investors the opportunity to either buy or sell default protection on a reference entity. The protection buyer pays a premium periodically for the possibility to get compensation if there is a credit event on the reference entity until maturity or the default time [4]. If there is a credit event the protection seller covers the losses by returning the par value. The spread of CDSs depends on the default probability of the underlying reference entity. In the literature, there are basically two main classes of credit risk models: reduced-form models and structural models [4]. In reduced-form models, the precise mechanism leading to default is left unspecified and the default time of a firm is modeled as a non-negative random variable, whose distribution typically depends on economic factors. Cariboni and Schoutens presented a structural model for credit derivatives where the firm asset value is considered as a case of Variance Gamma (VG) process [1]. A geometric form of the jump-diffusion process was applied in modeling the market value of a firm's asset to find a pricing formula for the spread [5]. Using the double exponential jump-diffusion model, the credit spreads and implied volatility of equity options was studied [3]. When a company defaults, the companys stock price inevitably drops by a sizeable amount. Carr and Wu have concluded that the possibility of default on a corporate bond generates negative skewness in the probability distribution of stock returns [2]. This negative skewness is manifested in the relative pricing of stock options across different strikes.
In this paper we are dealt with an intensity based model under a time-changed Levy process. The underlying

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distributions in the Levy models are very flexible and can take into account asymmetry and fat-tail behavior. Another advantages by following this approach is that the presence of jumps in the underlying process allows for instantaneous default. Time-changed processes with infinite activity are able to capture both frequent small moves and rare large moves, which makes them reasonable alternatives for jump diffusions when describing asset returns. Also, dealing with these processes allows one to calibrate models more quickly and describe dependence structures between assets in a more straightforward way. Time-changed Levy processes can capture asymmetry, leptokurtosis and thicker tail properties in returns in addition to characterizing persistence effect and heteroskedasticity effect in volatility. These processes subordinated by Ornstein-Uhlenbeck process incorporates jumps and stochastic volatility simultaneously. Moreover, we price a CDS spread, such that the expected premium payments on the CDS equate the expected loss payments. Finally, we calibrate the introduced model by a differential evolution algorithm that outperforms local search techniques to a series of realistic CDS.

2 Time-change Levy intensity process

On a filtered probability space $(\omega, \mathcal{F}, \{\mathcal{F}_t\}, Q)$, where Q is a risk-neutral measure, suppose that $Z_t = \sum_{i=1}^{N_t} X_i$ where $N = \{N_t; t \in \mathbb{R}, t > 0\}$ is a Poisson process with arrival rate $a \in \mathbb{R}^+$. Furthermore, $\{X_i\}_{i=1}^{\infty}$ is a sequence of independent identically distribution (iid) exponential random variables with mean $\frac{1}{b}$ for $b \in \mathbb{R}^+$. The instantaneous rate of activity process $y = \{y_t : t \in \mathbb{R}^+\}$ follows a non-Gaussian process by dynamic

$$dy_t = -\beta y_t dt + dZ_{\beta_t}, \tag{1}$$

subject to $y_0 \in \mathbb{R}$ and $\beta \in \mathbb{R}^+$. The measure Z is a subordinator Levy process that refers to back driving process. Due to the fact that Z has almost surely non-negative increments, the process y is strictly positive. To find the solution of stochastic differential equation (1), first we find the dynamics of $e^{\beta t} y_t$ that is $d(e^{\beta t y_t}) = e^{\beta t} dZ_{\beta t}$. Then by integrating both sides we obtain

$$y_t = e^{-\beta t} y_0 + \int_0^t e^{\beta(u-t)} dZ_{\beta u}.$$

The total activity process is $Y = \{Y_t; t \in \mathbb{R}^+\}$ where $Y_t = \int_0^t y_u du$. So, the characteristic function of Gamma OU process is

$$\phi(u) = exp\Big[iuY_0\beta^{-1}(1 - e^{-\beta t}) + \frac{\beta a}{iu - \beta b}\Big(bln(\frac{b}{b - iu\beta^{-1}(1 - e^{-\beta t})}) - iut\Big)\Big]. \tag{2}$$

Now we assume that the intensity process be a time-changed Levy $\lambda_t = X_{Y_t}$ such that the characteristic function of λ_t is presented by

$$\begin{aligned} E[e^{iuX_{Y_t}}] &= E[e^{Y_t \Psi_X(u)}] \\ &= E[e^{i(-i\psi_X(u))Y_t}] = \phi(-i\psi_X(u)), \end{aligned} \tag{3}$$

where $\psi_X(u)$ is the characteristic exponent of the Levy process X.
As a case, we consider the subordinated X a Variance Gamma (VG) process by the following characteristic function

$$\Psi_{VG} = (1 - iu\theta\nu + \frac{1}{2}u^2\sigma^2\nu)^{-\frac{t}{\nu}}. \tag{4}$$

Therefore,

$$\begin{aligned} \Phi(-i\psi_{VG}(u)) =& \Phi(-i[-\frac{1}{\nu}log(1 - iu\theta\nu + \frac{1}{2}U^2\sigma^2\nu)]) \\ &= \Phi(\frac{i}{\nu}log(1 - i\theta\nu + \frac{1}{2}u^2)\sigma^2\nu)), \end{aligned} \tag{5}$$

is the corresponding characteristic function of the intensity process.

2.1 Survival probability

In this part, first we present how to find the survival probability based on the intensity process. Then for the VG process we have detailed the calculations of the survival probability. Let X be a continuous-time with state space $\mathbb{R}^+$ and cumulative distribution function $F(x)$.
In survival analysis, one is more interested in the probability of an individual to survive to time x, which is given by the survival function

$$S(x) = 1 - F(x) = P(X \geq x) = \int_x^\infty f(s)ds.$$

The major notion in this study is the intensity function $\lambda(\cdot)$, which is defined by

$$\lambda(x) = lim_{\Delta \longrightarrow 0} \frac{P(x \leq X < x + \Delta \mid X \geq x)}{\Delta} = \frac{f(x)}{1 - F(x)}. \tag{6}$$

The intensity function characterizes the risk of dying changing over time or age. It specifies the instantaneous failure rate at time x, given that the individual survives until x. Some times, it is useful to deal with the cumulative (or integrated)

$$\Lambda(x) = \int_0^x \lambda(s)ds,$$

implies that

$$\Lambda(x) = \int_0^x \lambda(s)ds = \int_0^x \frac{f(s)}{1 - F(s)}ds = -ln(1 - F(x)),$$

and consequently

$$S(x) = 1 - F(x) = e^{-\int_0^x \lambda(s)ds} = e^{-\Lambda(x)}. \tag{7}$$

Intensity models assume that, an event of default occurs at the first jump of a counting process $M = \{M_t, t \geq 0\}$. The intensity rate $\lambda = \{\lambda_t, t \geq 0\}$, Known also as intensity time-changed model, represents the instantaneous default probability. Let τ be the default time, then the intensity of default is defined by

$$\lambda(t) = lim_{h \longrightarrow 0} \frac{P[\tau \in (t, t + h] \mid \tau > t]}{h}.$$

So τ is defined as the first jump of M_t as

$$\tau = \inf\{t \in \mathbb{R}^+ \mid M_t > 0\}.$$

The implied survival probability $P(t)$ is given by

$$\begin{aligned} P(t) = P[M_t = 0] = P[\tau > t] &= \mathbb{E}[\exp(-\int_0^t \lambda(s)ds)] \\ &= \mathbb{E}[\exp(-Y_t)] \end{aligned} \tag{8}$$

where $Y_t = \int_0^t \lambda(s)ds$ is the integrated of the intensity.
For the time-changed Levy intensity model, we obtain the following closed form solutions for the survival probabilities

$$\begin{aligned} \delta(t) = P(t) &= \mathbb{E}[\exp(-\int_0^t \lambda(s)ds)] \\ &= \Phi(-i\Psi_{VG}(i)) \end{aligned} \tag{9}$$

2.2 CDS spread formula

The CDS spread is set at inception, so that the contract is costless to enter. We price a CDS and the corresponding spread, such that the expected premium payments on the CDS equate the expected loss payments. Consider a CDS with maturity T and a continuous spread c. The value of the CDS is then given by

$$(1-R)N\Big(-\int_0^T e^{-rt}d\delta(t) - cN\int_0^T e^{-rt}\delta(t)dt\Big),$$

where the first term and the second term respectively correspond to the present value of the so-called loss leg and premium leg of the CDS contract. Moreover, $\delta(t)$ is the so called survival probability defined in (9). Pricing the CDS is to find the par spread $c = c(T)$ which makes the loss leg equal to the premium leg

$$c(T) = \frac{(1-R)\big(-\int_0^T e^{-rt}d\delta(t)\big)}{\int_0^T e^{-rt}\delta(t)dt} = (1-R)\Big(\frac{1-e^{-rT\delta(t)}}{\int_0^T e^{-rt}\delta(t)dt} - r\Big) \tag{10}$$

where R is the asset specific recovery rate and r is the default-free discount rate.

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