Designing Sound Deposit Insurances

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Abstract

Deposit insurances were blamed for encouraging the excessive risk-taking behavior during the 2008 financial crisis. The main reason for this destructive behavior was "moral hazard risk", usually caused by inappropriate insurance policies. While this concept is known and well-studied for ordinary insurance contracts, yet needs to be further investigated for insurances on financial positions. In this paper, we set up a simple theoretical framework for a bank that buys an insurance policy to protect its position against market losses. The main objective is to find the optimal insurance contract that does not produce the risk of moral hazard, while keeping the bank's position solvent. In a general setup we observe that an optimal policy is a multi-layer policy. In particular, we obtain a closed form solution for the optimal insurance contracts when a bank measures its risk by either Value at Risk or Conditional Value at Risk. We show the optimal solutions for these two cases are two-layer policies.

Keywords: Deposit insurance, solvency, risk measure and premium, Black-Scholes model, moral hazard, VaR, CVaR, stop-loss

Mathematics Subject Classification [2010]: G11, G13, G22

1 Introduction

An important lesson from the 2008 financial crisis is that an underestimated moral hazard risk can be destructive. Whilst this fact was widely known in insurance, it is rather new in the banking industry. Moral hazard is a phenomenon in which some agents take excessive risk because they know that the potential costs of taking further risk will be covered by other agents and/or the government.

An extensive use of deposit insurances in the financial sector caused excessive risk-taking behavior. In the years prior to the 2008 financial crisis, financial institutions, like banks and in particular hedge funds, bought deposit insurances in order to protect their investments. As a result, financial institutions could venture riskier investments, by transferring the big losses to the insurance companies; see [5], [7] and [8]. This was a reason for huge risk-taking behaviors which caused big losses in 2008.

In a banking system, moral hazard is a result of the absence of enough prudential policies. While, the minimum capital requirement is aimed to partly prevent the excessive risk-taking behavior by putting banks equity at risk, it can also encourage further risk-taking behavior. After 2008, it is proven that neither these measures nor any other prudential regulatory law can prevent another crisis unless the excessive risk-taking behavior is controlled, see [6].

The key to prevent the risk of moral hazard is that the financial system must not be used for reckless gambling inspired by excessive risk-taking behavior. In general, there are two methods to reduce the excessive risk-taking behavior. First, introducing ex-ante policies which enforce banks to bear part of any loss they impose to the system, and second, introducing ex-post policies which penalizes the excessive risky behaviors.

In this paper, we have chosen to set an ex-ante policy. In this approach, the risk of moral hazard is reduced by setting contracts that both the insurer and the insuree are being made partially responsible for the

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losses. We consider a bank that seeks an optimal insurance contract that does not produce any risk of moral hazard, while also keeping the bank's position solvent. By adopting a complete market model as in [9], where the author treated a deposit contract as an option, we will characterize the optimal contracts. Ultimately, we use α-percent Value at Risk and Conditional Value at Risk for the minimum capital requirement[2], and we observe that the optimal contracts are two-layer polices whose upper and lower retention levels are completely determined.

Our work is important from two perspectives: first, we introduce a mathematical framework to design deposit insurances that cannot impose the risk of moral hazard to the financial system. Second, we use actuarial mathematics methods that are rather new in related finance and banking problems. A similar approach is applied in [2], where a risk management under prudential policies is discussed. The problem of insurance and re-insurance design with no risk of moral hazard is very well studied in the literature of actuarial science, for instance in [4] and more recently in [1].

The rest of the paper is organized as follows: Section 2 introduces some mathematical notion and a model set-up for a bank balance sheet. In Section 3, general optimal solutions are discussed.

2 Model Set-up

Let $(\Omega, \mathbb{P}, \mathcal{F})$ be a complete probability space, where Ω is the set of all scenarios, $\mathbb{P}$ is the physical probability measure, and $\mathcal{F}$ is a σ- field of measurable subsets of Ω. We denote the set of all random variables by $L^0 = L^0(\Omega, \mathcal{F})$. Furthermore, $\mathbb{E}$ represents the mathematical expectation with respect to $\mathbb{P}$.

In this paper, we assume that contracts (policies) are issued at $t = 0$, let say the beginning of a policy year, and liabilities are settled at $t = T$, the maturity time or the end of the year. A random variable represents losses of a policy and for any $X \in L^0$, the cumulative distribution function associated with X is denoted by F_X. The constant risk-free interest rate is denoted by $r \geq 0$.

Let us consider a bank with an initial capital[3] $e^{-rT}b$, and a non-negative loss random variable $\mathcal{L} \geq 0$ at time T. By buying an optimal insurance contract, the bank wants to hedge its global position by transferring part of its losses to an insurance company. If we denote the insurance policy by a non-negative random variable I at time T, it has to satisfy $0 \leq I \leq \mathcal{L}$. The value of the insurance policy is given by a premium function $\pi : \mathcal{D} \to \mathbb{R}$ at time 0, where $\mathcal{D} \subseteq L^0$ is the domain of π. Therefore, the bank's global loss position is composed of four parts: the initial capital at time 0 i.e., $e^{-rT}b$, the global loss i.e., $\mathcal{L}$, the insurance policy i.e., $-I$, and the premium payed for the insurance policies at time 0, i.e., $\pi(I)$ which accumulates to $e^{rT}\pi(I)$ at time T. Therefore, a simplified balance sheet of the bank's position at time T is given as follows

Equity	Liability	Total Balance	Total Loss
$b + I - e^{rT}\pi(I)$	$-\mathcal{L}$	$b + I - e^{rT}\pi(I) - \mathcal{L}$	$e^{rT}\pi(I) + \mathcal{L} - b - I$

Table 1: The bank's balance sheet at time T.

The bank is solvent if its global position is solvent. To measure the solvency we use a risk measure; for instance, Value at Risk (VaR) or Conditional Value at Risk (CVaR). We recall that VaR is recommended in the Basel II accord for the banking system and also in the Solvency II for the insurance industry. In this paper, ϱ denotes the risk measure recommended by regulator. The bank is solvent if its capital b is adequate for the solvency i.e., $\varrho\left(e^{rT}\pi(I) + \mathcal{L} - b - I\right) \leq 0$. In other words, the position $e^{rT}\pi(I) + \mathcal{L} - b - I$ does not produces any risk. Therefore, an optimal decision for the bank is to buy the cheapest insurance contract i.e.,

$$\begin{cases} \min \pi(I), \\ \varrho(e^{rT}\pi(I) + \mathcal{L} - b - I) \leq 0, \\ 0 \leq I \leq \mathcal{L}. \end{cases} \tag{1}$$

[2] As recommended in the Basel II accord and Solvency II

[3] For technical reasons, we assume the value of b at time T and discount it to make it comparable to today's value.

Next, we use a more specific model for the bank's asset. Our paper follows an approach similar to [9], assuming that the market is composed of a risk-free asset and the risky asset that is modeled by a geometric Brownian motion. If the asset price at time $t \in [0,T]$ is denoted by S_t, then we assume that it follows:

$$\begin{cases} dS_t = \mu S_t dt + \sigma S_t dW_t, \\ S_0 > 0, \end{cases}$$

where W_t is a standard Wiener process, and the constants μ and σ are respectively the drift and volatility.

We suppose that the bank's loss is a non-negative and non-increasing function of its assets value. In mathematical terms, $\mathcal{L} = L(S_T)$, where $L : \mathbb{R} \to \mathbb{R}_+ \cup \{0\}$ is a non-increasing function. A natural example is losses due to negative returns

$$L_n(x) = \begin{cases} e^{rT} S_0 - x, & \text{if } x \leq e^{rT} S_0, \\ 0, & \text{if } x > e^{rT} S_0. \end{cases} \tag{2}$$

Assumption 1. We assume that there is no risk of moral hazard. If we suppose that both the bank and insurance loss random variables are non-decreasing functions of the global loss variable, then the risk of moral hazard is ruled out, as both sides are exposed to any increase in the global loss, see for example [3]. Therefore we assume that $I = f(\mathcal{L})$ where both f and $\mathrm{id} - f$ are non-decreasing (here id denotes the identity function).

Definition 2.1. A distortion risk measure ϱ_Π (or simply ϱ) is a mapping from $\mathcal{D}_\Pi$ to $\mathbb{R}$ defined as

$$\varrho_\Pi(X) = \int_0^1 \mathrm{VaR}_t(X) d\Pi_\varrho(t). \tag{3}$$

where $\Pi_\varrho : [0,1] \to [0,1]$ is a non-decreasing and cádlág function such that $\Pi(0) = 1 - \Pi(1) = 0$.

Assumption 2. We suppose that ϱ is a distortion risk measure and it satisfies the following regularity condition

$$\lim_{n\to\infty} \varrho(X \wedge n) = \varrho(X) \text{ for all random variables } X \text{ in } L^0. \tag{4}$$

The main results are presented in the next section, but first, we need to introduce some notation. Let $B = b - \varrho(\mathcal{L})$, $\Phi^\varrho(t) := 1 - \Pi_\varrho(t)$, $\Phi^\varrho_\mathcal{L}(t) := \Phi^\varrho(F_\mathcal{L}(t))$, $\Phi^{\bar{\pi}}_\mathcal{L}(t) = N\left(\frac{(\mu-r)\sqrt{T}}{\sigma} + \frac{\log\left(\frac{L^{-1}(t)}{S_0}\right) - \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$ (N is standard normal distribution), and

$$\theta^* := \underset{\theta \geq 0}{\operatorname{argmin}} \left(\int_0^\infty \left(\Phi^{\bar{\pi}}_\mathcal{L}(t) + \theta \left(\Phi^{\bar{\pi}}_\mathcal{L}(t) - \Phi^\varrho_\mathcal{L}(t) \right) \right)_+ dt + B\theta \right),$$

note that here $(x)_+ = \max\{x, 0\}$.

3 Main results

Theorem 3.1. *If Assumptions 1 and 2 hold, and if $m = \mu - r \geq 0$, the optimal solution to* (1) *is given by $I = f^*(\mathcal{L})$, where $f^*(x) = \int_0^x h^*(t)dt$, and*

1. *If $\theta^* > 0$*

$$h^*(t) = \begin{cases} 1, & \text{if } \Phi^{\bar{\pi}}_\mathcal{L}(t) < \frac{\theta^*}{1+\theta^*} \Phi^\varrho_\mathcal{L}(t), \\ 0, & \text{if } \Phi^{\bar{\pi}}_\mathcal{L}(t) > \frac{\theta^*}{1+\theta^*} \Phi^\varrho_\mathcal{L}(t), \end{cases}$$

$$\text{and } \int_0^\infty \left(\Phi^{\bar{\pi}}_\mathcal{L}(t) - \Phi^\varrho_\mathcal{L}(t) \right) h^*(t)dt = B,$$

2. If $\theta^* = 0$

$$h^*(t) = 0.$$

Corollary 3.2. *If in addition to Assumptions 1, 2 and* $m = \mu - r \geq 0$*, the following condition holds*

$$\forall \theta \geq 0, \left\{ 0 \leq t < \operatorname{esssup}(\mathcal{L}) \,|\, \Phi_{\mathcal{L}}^{\bar{\pi}}(t) = \frac{\theta}{1+\theta} \Phi_{\mathcal{L}}^{\varrho}(t) \right\} \text{ is of Lebesgue measure zero,}$$

then the optimal solution to problem (1) *is given by*

$$h^* = 1_{\left\{ \Phi_{\mathcal{L}}^{\bar{\pi}} < \frac{\theta^*}{1+\theta^*} \Phi_{\mathcal{L}}^{\varrho} \right\}}.$$

The next theorem shows that for the particular risk measure VaR_α, the optimal deposit insurances are indeed stop-loss policies. A similar result can be obtained for CVaR_α.

Theorem 3.3. *If* $\varrho = \mathrm{VaR}_\alpha$*,* $m = \mu - r \geq 0$*, and the assumptions of Corollary 3.2 hold, then the optimal insurance contract* I *is a two-layer policy with upper retention level* $u = F_{\mathcal{L}}^{-1}(\alpha) = \mathrm{VaR}_\alpha(\mathcal{L})$ *and a lower retention level* l *given as a solution to*

$$\bar{\pi}(\min\{\mathcal{L} - l, 0\}) + b = \bar{\pi}(\min\{\mathcal{L}, \mathrm{VaR}_\alpha(\mathcal{L})\}). \tag{5}$$

Remark 3.4. In contrast with [9], where it is assumed that an insurance contract is a put option, our assumptions lead to the contracts that are bounded from above.

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