Counterparty Risk

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Abstract

Counterparty credit risk is the combination of market risk and credit risk. The Credit Valuation Adjustment (CVA) has become an important part of Basel III, that used for measuring counterparty risk. Wrong way risk is one of the factor that affects the amount CVA. Wrong way risk refers positive dependent between market risk and credit risk. In this paper, we present a method, for bounding the impact of wrong-way risk on CVA. We simulate the market exposure and default time and then we find the worst case CVA resulting from wrong way risk by solving linear program.

Keywords: Credit Valuation Adjustment, Counterparty Credit Risk, Wrong-way Risk, Iterative Proportional Fitting Process (IPFP)

1 Introduction

As a firm enters into a swap contract, it is exposed to market risk through changes in market prices that affect the cash flows. It is also exposed to the risk that the party on the other side of the contract may default and fail to make payments due to the transaction. The risk of this contract type is called **counterparty risk**. Counterparty credit risk (often known just as counterparty risk) can be defined as the risk that a counterparty in a financial contract might not be able or willing to full the obligations on other side of the contract. The counterparty risk is closely related to the traditional credit risk, which can be regarded as lending risk. This type of risk concerns loans, bonds, mortgages and so on. However, compared to credit risk there are two sides to counterparty credit risk. Former market risk that determinies the magnitude of one party's exposure to another party, and later, bilateral credit risk. Counterparty risk is arising from two broad classes of financial products: the first, OTC (over-the-counter) derivatives, for example interest rate swaps, credit default swaps and FX forwards, the second securities financing transactions, for example repos and reverse repos and securities borrowing and lending.

The standard tool for quantifying counterparty risk is the **Credit Valuation Adjustment (CVA)**. In fact, CVA is the difference between the portfolio's risk-free value and the portfolio's true value, taking into account the possibility of credit deterioration of the counterparty. In the calculation of CVA, it is usually assumed that the counterpartys probability of default is independent of the dealers exposure. However, practically, there is a dependence between exposure and credit quality. That positive (Negative) dependence known as wrong way (right-way) risk, in fact, the risk is called wrong way (right way) if exposure tends to increase (decrease) when counterparty credit quality worsens. The CVA for a portfolio of derivatives will generally increase with wrong-way risk increaseing, but the correct degree of wrong-way risk is difficult to estimate in practice. In this paper, we introduced a method for bounding wrong-way risk by finding largest CVA. We use a standard simulation for CVA calculation by simulation of market factors to achieve exposure at any moment and default time too. Based on the simulation of market exposure and counterparty's time to default, we find the worst case CVA by solving a linear porogramming problem. The worst case can be overly conservative, so we extend the procedure by adding penalty term in the objective function. The models that to describe dependence between exposure and probability of default in calculation CVA are found Hull and White [3], Rosen and Saunders [2], Brigo, Morini, and Pallavicini [4].

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2 Main results

Let τ to be the time at which a counterparty defaults and R denote recovery rate. Moreover $V^{+}(\tau)$ denote the positive exposure at the time of its default, discounted to time zero. The CVA for a time horizon T defined by

$$CVA = \mathrm{E}[(1-R)V^{+}(\tau)\, 1_{\{\tau<T\}}].$$

The CVA is usually calculated over a fixed set of dates. So set $0 = t_0 < t_1 < ... < t_d = T < t_{d+1} = \infty$. Then we generate positive exposure paths $(V^{+}(t_1), ..., V^{+}(t_d))$ by simulation of market risk factors. The market risk model determines the law of $(V^{+}(t_1), ..., V^{+}(t_d))$ by a probability measure p on $\mathbb{R}^d$. We suppose the probability that default occurs at t_k is q_k, $k = 1.....,d$. Now we let

$$X = (V^{+}(t_1), V^{+}(t_2), ...V^{+}(t_d)), \quad Y = (1_{\{\tau=t_1\}}, ..., 1_{\{\tau=t_d\}}).$$

In the sequel, worst-case CVA defined by

$$\sup_{\mu\in\Pi(p,q)} \int_{\mathbb{R}^d\times\mathbb{R}^d} < x, y > d\mu(x, y),$$

in which $\prod(p, q)$ denotes the set of probability measures on $\mathbb{R}^d \times \mathbb{R}^d$ with marginals p and q. If the joint law for X and Y are unknown, then we can estimation worst-case CVA by simulation as follows. Let $X_1, X_2, ...X_N$ be N independent copies of X and $Y_1, Y_2, ...Y_N$ be N independent copies of Y, and, pN, and qN denote empirical measures for sets X and Y in $\mathbb{R}^d$. We can find the worst-case joint mass function P_{ij} on the set of pairs $\{(X_i, y_j);\, i = 1...N, j = 1...d+1\}$. We estimate worst-case CVA by

$$CVA = \max_{\{P_{ij}\}} \quad \sum_{i=1}^{N}\sum_{j=1}^{d+1} C_{ij}P_{ij},$$

$$\sum_{j=1}^{d+1} P_{ij} = \frac{1}{N}, \quad i = 1, ...N,$$

$$\sum_{i=1}^{N} P_{ij} = qN(y_j), \quad j = 1, ..d+1,$$

$$P_{ij} \geq 0, \quad i = 1...N, \quad j = 1...d+1,$$

where $C_{ij} =< X_i, y_j >$ and $y_1 = (1, 0, ..., 0), ...y_d = (0, 0, ...1), y_{d+1} = (0, 0, ..., 0)$.
The worst-case joint distribution is useful because it provides a bound but it can be overly pessimistic, so we extend the procedure by adding penalty term in the objective function. For this, we choice **Relative Entropy** which is also known as Kullback-Leibler divergence. For probability measures P and F on a common measurable space with $F >> P$, the relative entropy of P to F is defined as

$$D(P|F) = \int \ln(\frac{dP}{dF})dP.$$

Relative entropy as a function of P is nonnegative, also $D(P|F)$ is convex in P and leads to a convex optimization problem. Hence, our problem by applying the penalty term is given as

$$\max_{\{P_{ij}\}} \sum_{i=1}^{N}\sum_{j=1}^{d+1} C_{ij}P_{ij} - \frac{1}{\theta}\sum_{i=1}^{N}\sum_{j=1}^{d+1} P_{ij}\ln(\frac{P_{ij}}{F_{ij}}),$$

in which F_{ij} denote the joint probability in the independent case, these are given by $F_{ij} = qN(y_j)/N$, $i = 1, ...N$, $j = 1, ...d+1$, The problem can be solved through a simple algorithm known as the **Iterative Proportional Fitting Procedure (IPFP)**. Indeed IPFP is a mathematical procedure for adjusting a table of data cells such that marginal totals remain fixed. The IPFP method takes as input data a nonnegative matrix and row and column marginals.

References

[1] Glasserman, P., and Yang, L. (2015). *Bounding wrong-way risk in measuring counterparty risk.* working Paper, Office of Financial Research and Columbia University.

[2] Rosen, D., and Saunders, D. (2012). *CVA the wrong way.* Journal of Risk Management in Financial Institutions 5.3 : 252-272.

[3] Hull, J., and Alan, W. (2012). *CVA and wrong-way risk.* Financial Analysts Journal 68.5 : 58.

[4] Brigo, D., and Morini, M., and Pallavicini, A. (2013). Counterparty credit risk, collateral and funding: with pricing cases for all asset classes. John Wiley and Sons.

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