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Systemic Risk Measurement in a Banking Network Using Conditional VaR and Vine Copula

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Abstract

Investors and regulators have paid more attention to the the sensitivity of the financial system since recent financial crises (the collapse of Lehman Brothers and the European sovereign debt crisis in 2008). A bank's default role in systemic risk reveals and shows its importance to manage and control this kind of risk because of its destructive effects on financial system. Using mathematical and statistical tools, we measure the systemic impact of financial distress among simulated bank return data, and empirical data from some Iranian banks using conditional value at risk (CoVaR) as a systemic risk measure. In fact, it is important to know how the fragile position of one financial institution could impair the performance of other financial institutions. We model multivariate dependence between simulated data and empirical Iranian banks data using a hierarchical tree structure given by a vine copula, the dependence between and among these banks is identified. For about 1000 points in time for simulated data and from 2009 to 2018 for Iranian banks, we determine the increases and decreases in systemic risk. It is identified if any of them has a predominant or minor role in risk transmission. These results have implications for the regulation of capital in financial institutions and for investor's risk management decisions.

Keywords: Systemic Risk, ΔCoVaR Measure, Vine Copula, MVGARCH Models
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1 Introduction

When we are studying a particular institution's risk, we are faced with different kinds of effective risks like credit risk, operational risk, liquidity risk and etc. On the other hand, studying risks that a financial system is involved with, reveals systemic risk. Any of the risks mentioned above may be the source of systemic risk. In fact, systemic risk implied by a bank default can trigger a domino effect. Before and after the recent financial crises different set of financial instruments with various approach emerged and researchers try to find better ways to measure systemic risk in order to manage and control its domino effects in whole financial system. We use the notion of Conditional Value-at-Risk (CoVaR) measure introduced by Adrian and Brunnermeier (2011) as a dependence adjusted version of Value-at-Risk (VaR) and use only publicly available information.

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Definition 1.1. The CoVaR of a financial institution is the VaR of the financial institution conditional on the fact that another financial institution is in financial distress. Let X_t^1 be the returns of bank 1 and X_t^2 be the returns of bank 2. The CoVaR of bank 1 for a confidence level $1-\beta$ and time t can be formally defined as the β-quantile of the conditional distribution of X_t^1 :

$$Pr[X_t^1 \leq CoVaR_{\beta,t}^{1|2} | X_t^2 \leq VaR_{\alpha,t}] = \beta, \tag{1}$$

where $X_t^2 \leq VaR_{t,\alpha}^2$ means that bank 2 is in financial distress, where $VaR_{t,\alpha}^2$ is the VaR for bank 2, measuring the maximum loss that bank 2 may experience for a confidence level $1-\alpha$ and a specific time horizon t, that is, the α-quantile of the return distribution for bank 2: $pr(X_t^2 \leq VaR_{t,\alpha}^2) = \alpha$.

According to equation (1) we need information on the bivariate joint dependence between X_t^1 and X_t^2, and to determine second round effects on X_t^1 and so on we need multivariat dependence between all the financial institutions in the financial system. To achieve this aim we contribute Sklar's theorem. So, we can rewrite the equation (1) as :

$$C_{X_t^1,X_t^2}\big(F_{X_t^1}(CoVaR_{t,\beta}^{1|2}), F_{X_t^2}(VaR_{t,\alpha}^2)\big) = \alpha\beta. \tag{2}$$

We express dependence with copula functions. More specifically, we use vine copulas to find direct and indirect dependence among banks in the banking network. Also, which bank plays a pivotal role in systemic risk is identified. Systemic risk measure through CoVaR approach contains three steps bellow:

- **Step 1:** We compute the cumulative probability for the CoVaR, $u = F_{x_t^1}(CoVaR_{t,\beta}^{1|2})$ by solving from Equation 2.
- **Step 2:** From u, we invert the marginal distribution function of X_t^1 to obtain the CoVaR, hence, $CoVaR_{t,\beta}^{1|2} = F_{x_t^1}^{-1}(u)$.
- **Step 3:** We compute $\Delta CoVaR$. The systemic risk contribution of bank 2 can be defined as the difference between the CoVaR for a confidence level $1-\beta$ and the CoVaR of bank 1 conditional on the fact that bank 2 is in a benchmark state (the VaR value of $\alpha = 0.5$). This measure, called delta CoVaR ($\Delta CoVaR$), is defined as:

$$\Delta CoVaR_{\alpha,\beta} = CoVaR_{\alpha,\beta} - CoVaR_{0.5,\beta}. \tag{3}$$

2 Methodology

Taking into account Sklar's theorem to achieve CoVaR value we require specification of the copula function and of the marginal distribution of bank returns. Consider y_t is a financial return at time t and is calculated as $y_t = \ln \frac{p_t}{p_{t-1}}$, where p_t is the price of bank stock at time t. Then, y_t will be modelled as:

$$y_t = \mu_t + \varepsilon_t \tag{4}$$

$$\varepsilon_t = h_t^{1/2} z_t \tag{5}$$

where μ_t describes the conditional mean ($E\{y_t|F_{t-1}\} = \mu_t$). So, we modelled the returns for each bank through an ARMA(p,q), specified as:

$$X_t^i = \phi_0 + \sum_{j=1}^{p} \phi_j X_{t-j}^i + \sum_{h=1}^{q} \varphi_h \varepsilon_{t-h}^i + \varepsilon_t^i, \tag{6}$$

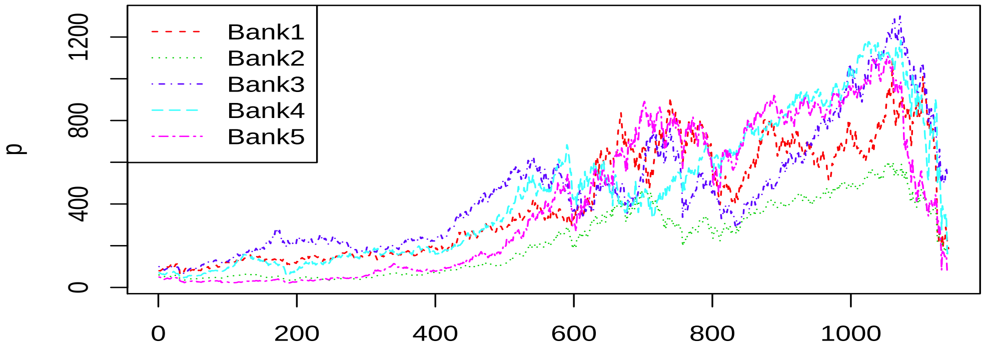

Figure 1: Price time series

Table 1: C-Vine parameter Estimation

tree	edge	copula	par 1	par 2	τ -kendal	upper-tail dependence	lower-tail dependence
1	2,3	survival Clayton-Gumbel	0.08	1.27	0.24	0.56	0.22
	2,4	t	0.58	4.58	0.44	0.27	0.27
	2,1	survival Clayton-Gumbel	0.09	1.26	0.24	0.58	0.27
	5,2	Clayton-Gumbel	0.41	1.47	0.44	0.40	0.32
2	2—1,5	t	0.13	8.34	0.08	0.024	0.024
	2—5,3	t	0.15	11.24	0.09	0.01	0.01
	2—5,4	t	0.40	5.42	0.26	0.4	0.4
3	5,2— 3,1	Clayton-Gumbel	0.2	1.04	0.13	0.065	0.03
	5,2—4,3	survival Clayton	0.07	0	0.03	0.00001	0
4	5,3,2—4,1	t	0.04	12.89	0.03	0.003	0.003

where p and q are respectively AR and MA orders. In equation (5) which specifies the innovation ε_t , where h_t is the conditional variance ($E\{y_t^2|F_{t-1}\} = h_t$) whose dynamic is reflected in a threshold generalised autoregressive conditional heteroscedasticity (TGARCH) specification (Zakoian (1994)) is given by

$$h_t^{1/2} = \alpha_0 + \alpha_1|\varepsilon_{t-1}| + \gamma I\{\varepsilon_{t-1} < 0\}|\varepsilon_{t-1}| + \beta_1 h_{t-1}^{1/2}, \tag{7}$$

where α_0 is a constant, h_{t-1} is the previous period's variance, ε_{t-1} represents the volatility shock for the previous period and I is an indicator function. γ captures leverage effects: thus, when it takes values greater than zero, the future conditional variance will increase proportionally more after a negative shock that after a positive shock of the same magnitude. Finally, z_t is an i.i.d. process with zero mean and unit variance that follows a Hansen (1994) skewed-t density distribution given by

$$f(z_{i,t};\nu,\eta) = \begin{cases} bc\Big(1 + \frac{1}{\eta-2}\big(\frac{bz_{i,t}+a}{1-\eta}\big)^2\Big)^{-(\nu+1)/2} & z_{i,t} < -a/b \\ bc\Big(1 + \frac{1}{\eta-2}\big(\frac{bz_{i,t}+a}{1+\eta}\big)^2\Big)^{-(\nu+1)/2} & z_{i,t} \geq -a/b \end{cases} \tag{8}$$

where $a = 4\eta c\big(\frac{\nu-2}{\nu-1}\big)$, $b^2 = 1 + 3\eta^2 - a^2$, $c = \Gamma(\frac{\nu+1}{2})/\sqrt{\pi(\nu-2)\Gamma(\frac{\nu}{2})}$. The parameters ν and η are the degree of freedom$(2 < \nu < \infty)$, and the symmetric parameter $(-1 < \eta < 1)$, respectively.

As we need to show multivariate dependence and time-varying behaviour of financial time series we use

Table 2: Systemic risk measurement

Bank	$CoVaR_{\alpha,\beta}$	$CoVaR_{0.5,\beta}$	$\Delta CoVaR_{\alpha,\beta}$
2,1	-3.88	-2.32	-1.56
2,4	-2.28	-1.43	-0.85
2,5	-4.65	-2.56	-2.09
2,3	-3.70	-2.25	-1.45
5,1	-1.85	-1.79	-0.067
5,3	-1.92	-1.78	-0.14
5,4	-2.165	-1.63	-0.53
3,1	-3.18	-2.20	-0.98
3,4	-2.05	-2.03	-0.02
4,1	-1.804	1-.85	-0.05

MVGARCH model introduced by Bollerslev, Engle, and Wooldrige (1988) given by

$$\mathrm{vech}(\mathbf{H}_t) = \mathbf{A}_0 + \sum_{i=1}^{P} \mathbf{A}_i \mathrm{vech}(\boldsymbol{\varepsilon}_{t-i}\boldsymbol{\varepsilon}'_{t-i}) + \sum_{j=1}^{Q} \mathbf{B}_j \mathrm{vech}(\mathbf{H}_{t-j}). \tag{9}$$

We use Dynamic Conditional Correlation (DCC) developed by Engle (2002). The DCC model tries to shape possible time-varying behavior of the correlation between time series by a GARCH-based procedure. We use Heinen and Valdesogo Approach (2009) in time-varying vine copulas. The correlation coefficient that is achieved through Engles DCC-approach is mapped into the dependence parameter of the different copulas via the respective Kendalls tau through the equation below,

$$\tau_t = \frac{2}{\pi} \arcsin(\rho_t). \tag{10}$$

3 Main results

In this research we work on simulated data and empirical data for Iranian banks, but results for simulation come here.

Figure 1 shows the price time series for simulated data. Using vine copula type C to reach direct and indirect dependence among banks results tabel 1. Bank 2 has pivotal rol in the banking network systemic risk and its effects in first round comes to bank 5. The results for systemic risk measurement come in table 2. Bank 2 and Bank 5 are more risky ones as it shows the most negative amount in table 2 belongs to them.

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