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More Investigations on Fair Valuation of Insurance Liabilities by Mean-Variance Hedge Based Approach

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Abstract

Many of the modern insurance contracts, such as equity-linked and the universal life products, include at least one feature which is related to the financial markets. Therefore, in valuation of the liability of these type of products, the financial markets and their relations with products must be considered. For these kind of modern insurance contracts, Solvency II requires that insurance companies should apply a fair valuation of assets and liabilities, in such a way that any claim with financial-actuarial construction is evaluated by combining market-consistency and actuarial approach. In this talk, we overview fair valuations and in addition the class of hedge based valuations and also one case of this class which is known as mean-variance hedge based valuation in a single period framework introduced by [1]. An example of mean-variance hedge based valuation in a special case in which there exists dependence between mortality and stock is given and we see that emerging dependence (between the time-1 price of stock and survival index) can change the result of valuation.

Keywords: :Fair Valuation, Market-Consistent Valuation, Actuarial Valuation, Hedge Based Valuation, Mean-Variance Hedge Based Valuation

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1 Introduction

The relation between many of modern insurance contracts such as equity-linked products and financial markets in one hand and solvency II requirement to fair valuation of insurance liabilities in other hand motivate the researchers to develop traditional valuations techniques to modern ones such that consider both financial and actuarial risks together. In [1], the bases are adapted in a consistent way. Mortality risk and stock price variations usually are considered to be independent, however, the global economic conditions affects both of them. Therefore, the association of these two variables must be taken into account. Our new work in this field is considering possitive and negative associations between time-1 stock price and mortality. In addition, independence between time-1 stock price and mortality is also considered and a comparison is made.

In the following, we first present the necessary definitions and assumptions, then an example of mean-variance hedge based valuation is given.

[1]speaker

Definition 1.1. Hedging: A risk management strategy designed to reduce or offset price risks and mainly using derivative contracts, the most common of which are futures and options.

Here we adopt the definition of market-consistent value as explained in [2]

Definition 1.2. Market-Consistent Value: A market consistent value of an asset or liability is its market value, if it is readily traded on a market at the point in time that the valuation is struck, and, for any other asset or liability, a reasoned best estimate of what its market value would have been had it been readily traded at the relevant valuation point.

The following definitions are taken from [1]

Definition 1.3. Fair valuation: A fair valuation is a valuation that is both market-consistent and actuarial. In other words a fair valuation is a valuation in which any financial part of claim is valuated by its market price and remaining part of claim (may be independent of the time-1 asset prices) valuated by an actuarial approach.

Vectors of time-0 and time-1 asset prices are as follow, respectively:

$$\mathbf{y} = (y^{(0)}, y^{(1)}, ..., y^{(n)}); \qquad y^{(m)} > 0 \;\; for \;\; m = 1, 2, ..., n. \tag{1}$$

$$\mathbf{Y} = (Y^{(0)}, Y^{(1)}, ..., Y^{(n)}); \qquad Y^{(m)} \geq 0 \;\; for \;\; m = 1, 2, ..., n. \tag{2}$$

In addition asset 0 is a risk free zero coupon bond such that $y^{(0)} = 1$ and $Y^{(0)} = e^r$, where $r \geq 0$ is continuously compounded interest rate.

In order to determine hedge based value of claim S, one first splits this claim into a hedgeable claim, which (partially) replicates S, and a remaining claim. The value of the claim S is then defined as the sum of the financial price of the hedgeable claim and the value of the remaining claim, determined according to an actuarial valuation.

Definition 1.4. Hedge based valuation: The valuation $\rho : \mathcal{C} \longrightarrow \mathbb{R}$ is a hedge based valuation if for any claim S, the value $\rho[S]$ is determined by:

$$\rho[S] = \boldsymbol{\theta}_S.\mathbf{y} + \pi[S - \boldsymbol{\theta}_S.\mathbf{Y}]. \tag{3}$$

Where $\boldsymbol{\theta}$ is a fair hedger and π is an actuarial valuation.

We notice that the fair hedger is a trading strategy ((n+1)-dimensional real-valued vector) which is both actuarial and market-consistent.

Theorem 1.5. *[1] A valuation $\rho : \mathcal{C} \longrightarrow \mathbb{R}$ is a hedge based valuation if and only if it is a fair valuation.*

Definition 1.6. Mean-variance hedging: For any claim S, the mean-variance hedge $\boldsymbol{\theta}^{MV}$ is the hedger for which the $\mathbb{P}$-expected quadratic hedging error is minimized:

$$\boldsymbol{\theta}_S^{MV} = \arg\min_{\boldsymbol{\mu} \in \boldsymbol{\Theta}} E^{\mathbb{P}}[(S - \boldsymbol{\mu}.\mathbf{Y})^2]. \tag{4}$$

Definition 1.7. Mean-variance hedge based valuation: The valuation $\rho : \mathcal{C} \longrightarrow \mathbb{R}$ is a mean-variance hedge based valuation if for any claim S, the value $\rho[S]$ is determined by:

$$\rho[S] = \boldsymbol{\theta}_S^{MV}.\mathbf{y} + \pi[S - \boldsymbol{\theta}_S^{MV}.\mathbf{Y}]. \tag{5}$$

Where $\boldsymbol{\theta}^{MV}$ is a mean-variance hedger and π is an actuarial valuation.

2 Main results

In this section, we presente an example of mean-variance hedge based valuation in a special case in which there exists dependence between mortality and stock price. We see that emerging dependence (between the time-1 price of stock and survival index) can change the result of the valuation.

The example presented in three different cases which are independence, positive and negative association. each of these cases are investigate in three market situations. The results are as follow:

The purpose is valuation of claim $S = (1 - Y^{(1)}) \times (1 - \tau)$, where $Y^{(1)}$ is time-1 stock price and τ is time-1 survival index price. Where market situations (a), (b) and (c) are as follow:

Table 1: Mean-variance hedge based valuation

relation between $Y^{(1)}$ and τ	traded assets in market	mean-variance hedge	$\rho[S]$
independence	(a)	(0.54,-0.57)	0.27
	(b)	(0.71,-0.57,-0.43)	0.16
	(c)	(0.94,-1,-1,2)	0.11
positive association	(a)	(0.71,-0.75)	0.35
	(b)	(0.83,-0.53,-0.53)	0.22
	(c)	(0.94,-1,-1,2)	0.11
negative association	(a)	(0.27,-0.33)	0.15
	(b)	(0.61,-0.53,-0.47)	0.05
	(c)	(0.94,-1,-1,2)	0.11

(a): A zero coupon bond and a stock are traded in market.
(b): A zero coupon bond, a stock and a survival index are traded in market.
(c): A zero coupon bond, a stock, a survival index and a call option are traded in market.
As we see in the above table, in (c) because the market is complete, the mean-variance hedge based values of claim are equal in three cases (independence, positive and negative association), this is because of when the market is complete, the mean-variance hedge based value of claim (which is the fair value of claim), is market-consistent value of claim and since the market-consistent value of claim in a arbitrage free and complete market is uniqe, the mean-variance hedges and consequently mean-variance hedge based values are the same in all three cases.

Another point to be seen in the results, the mean-variance hedge based value of claim S is minimum under negative association assumption and is maximum under positive association assumption. these results are reasonable because of claim S construction.

References

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