The 5th FINACT-IRAN National Conference on Financial and Actuarial Mathematics
December 22-25, 2018

The Stochastic Conditional Duration Model: An Empirical Study in TSE

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Abstract

This study modeled the transaction and volume durations for three stocks listed in Tehran Stock Exchange (TSE). The findings indicated that stochastic conditional duration (SCD) model can properly model trade and volume duration. Further, comparing the SCD model and autoregressive conditional duration (ACD) model indicated that both models properly explains the durations. However, the SCD model outperformed ACD in regards to describing data overdispersion, high correlation, and predicting trade duration

Keywords: Trade duration, Volume duration, Autoregressive conditional duration (ACD), Stochastic conditional duration (SCD).
Mathematics Subject Classification [2018]: jel: c41, c58, g12

1 Introduction

Trade duration reflects information arrivals to the market and serves as an indicator for measuring transaction density. Volume duration is the time required for trading specific volume of shares and serves as an indicator for market liquidity. Engle and Russel (1998) introduced the autoregressive conditional duration (ACD) model for modeling time intervals between trades. One of the advantages of ACD model is factoring irregularly time-spaced feature of trades and clustering property of durations which could not be modeled with classical econometrics models.

Since its introduction, the ACD model and its various extensions have become a leading tool in modeling the behavior of irregularly time-spaced financial data. Numerous studies proved the nonlinearity of financial data, therefor some extensions of ACD model were proposed based on the nonlinearity of duration, leading to the threshold ACD (TACD) model where conditional intervals are nonlinearly dependent on previous information. Another popular ACD extensions are the fractional integrated ACD (FIACD) model and logarithmic- ACD model in order to prevent non-negative limitations and give more flexibility in introducing latent variables.

Based on the similarity between ACD and GARCH, Bauwens and Veredas (2004) proposed the stochastic conditional duration (SCD) model. Compared to the ACD model, the SCD model proposed by Bauwens and Veredas (2004) are based on the assumption that the evolution of the conditional duration is driven by a latent variable. The latent variable is used to show the information flow leading to transactions and change in prices and transaction volumes in the market. As the SCD model has not been empirically studied in Irans capital market, this study aims to model trade and volume duration via SCD model. Further, we compared the results with the ACD model. As SCD estimates usually are based on QML Gaussian maximum likelihood method which does not rely on real time intervals estimation and is an approximate method, the study compares the performance of Kalman Filter and also Thavaneswaran approach in estimating the SCD

model via QML.
Methodology
The research data consisted of sample transactional data for three listed shares in TSE (Shabandar, Khodroo, and Tipico) in a three-month interval (September to December 2016) including the time, volume, and price variables. In ACD (1,1) model, the duration (the waiting time between consecutive trades) at t is the multiplication product of a random variable ε_t and conditional expected duration at time t (Eq. 1). The conditional expectation of trade duration (ψ_t) at t is estimated according to conditional expected duration of previous trades and past durations (Eq. 2).

$$d_t = \psi_t \times \varepsilon_t \qquad (1)$$

$$\psi_t = \omega + \alpha_1 \, d_{t-1} + \beta_1 \, \psi_{t-1} \qquad (2)$$

Where ε_t , the standardized durations; $\varepsilon_t = \frac{d_t}{\psi_t}$, is an iid innovation with a given parametric density.
When log transformation is taken on both sides of the ACD model in Eq. 1, it results in additive form of the logarithmic conditional duration. This transformation relaxes the positivity restriction on the conditional expectation and motivates some other extensions of ACD using log duration like Bauwens and Giot (Bauwens and Giot, 2000). In this study we consider stochastic process for log duration, named SCD. The SCD model takes the following form (Bauwens and Veredas, 2004),
Observation equation $d_t = \psi_t \times \varepsilon_t$
The latent equation $ln\ \psi_t = \omega + \beta \ln \psi_{t-1} + u_t,\ |\beta < 1|$
The model has two sources of uncertainty, ε_t for unobserved duration and u_t for conditional duration.
$|\beta| < 1$ is the stationarity condition. In SCD model each distribution with a positive domain (due to the positive duration) can be used to error terms. QML maximum likelihood with Kalman filter (Bauwens and Veredas, 2004), the Monte Carlo Markov Chain, the empirical characteristic function and the GMM methods are the conventional methods for estimating SCD parameters. While the SCD model has a multiplicative specification similar to the ACD model, it differs from the latter because it is a double stochastic process. The conditional expected duration, which was a fixed function of unknown parameters under the ACD model, is assumed to be a random variable under the SCD model. One of the main difficulties in estimation SCD models, is the evaluation of the likelihood function for carrying out parametric inference, because the latent variable must be integrated out. Overall, the estimation approaches used in the literature are based on a transformation of the nonlinear time series model into a linear state space representation and application of the Gaussian Kalman filter. One major drawback is that these approaches neither address the information associated with the corresponding estimating functions nor the efficiency of the resulting estimates. The combined estimating functions approach based on generalized martingale estimating functions introduced by Thavaneswaran and his colleagues (Thavaneswaran et al, 2014) is used in this article. Finally, we used the mean absolute percentage of error (MAPE) to compare the performance of SCD and ACD models.

2 Main results

Results indicated that SCD model can properly explains transactional data attributes including clustering and correlation. Besides, it can be said that compared to the Kalman filter, Thavaneswaran filter has ability to capture higher correlation between the latent variable and duration. In addition, in Thavaneswaran approach for Khodroo and Shabandar, the estimated and actual coefficients of dispersion for the volume and trade durations were more similar compared to the coefficients of dispersion estimated via the Kalman filter.

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