Solving Parametric Fractional Differential Equations Arising from Rough Heston Model using Quasi-Linearization and Spectral Collocation

Maryam Vahid Dastgerdi, Ali Foroush Bastani
Department of Mathematics, Institute for Advanced Studies in Basic Sciences, Zanjan, Iran

Abstract
The rough Heston model has recently attracted the attention of many finance practitioners and researchers as it maintains the basic structure of the classical Heston model while having descriptive capabilities in terms of micro-structural foundations of the market. Using the fact that the characteristic function of log-price in this model could be expressed in terms of the solution of a nonlinear parametric fractional Riccati differential equation not admitting a closed-form solution, devising efficient numerical schemes for pricing and calibration under this model has become a crucial need in the computational finance community. Although the fractional Adams method has been used in most of the recent studies on the rough Heston model, this method suffers from some stability and convergence issues in treating the problem. In this paper, we present a numerical method based on Newton-Kantorovich quasi-linearization to solve the nonlinearity issue followed by spectral collocation based on “poly-fractonomials” to approximate the fractional derivatives in an accurate and efficient manner. We provide sufficient conditions under which our method is convergent and the order of convergence is also obtained. In order to guarantee the specified convergence rate, we first prove some regularity results on the linearized problem and then employ the proposed scheme to solve a practical calibration problem from the SPX options market. The efficiency of the proposed method is illustrated by comparing the results with the fractional Adams method.

Keywords: Rough Heston model, Fractional nonlinear Riccati differential equation, Spectral collocation,
Newton-Kantorovich quasi-linearization.
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